Pre-class Warm-up!!

A particle moves along a path $c(t) = (t^3 - 3, 2t^2 + 1)$

What is the speed of the particle at $t = 1$?

a. 3 units /sec

b. 4 units /sec

c. 5 units /sec ✓

d. √ 5 units /sec

e. None of the above.

$c'(t) = (3t^2, 4t)$

$c'(1) = (3, 4)$

$\|c'(1)\| = \sqrt{3^2 + 4^2}$

$= 5$

4.1 Paths again. Acceleration and Newton's Second Law

We recall:

- a path is a mapping c : [a,b] -> R^n
- we can differentiate it to get a velocity vector v = c′(t)
- it satisfies some rules: sum rule, scalar multiplication rule, chain rule and NEW dot product rule, cross product rule
- Also new: acceleration vector. $a = v'(t) = c''(t)$
- Newton: force = mass x acceleration

Terminology I will not use: regular path.

A regular path has $c'(t) \neq 0$

$c(t) = t^3$ is not regular.

Typical HW questions:
Find the velocity and acceleration vectors

Verify the rules.

Given values of c″(t), c′(0) and c(0) find c.

Find the force on a particle under some given acceleration.

Rules: 1. $(ac+bd)' = ac' + bd'$.

3. $\frac{d}{dt}(c \cdot d) = c' \cdot d + c \cdot d'$

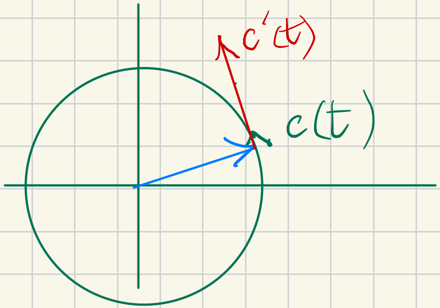
4. If $c, d : \mathbb{R} \to \mathbb{R}^3$ then

$(c \times d)' = c' \times d + c \times d'$

2. Chain rule.

Examples:

1. (Like qn 20) If $\| c(t) \|$ is constant then $c'(t)$ is perpendicular to $c(t)$ for all t.



Solution. $c(t) \cdot c(t) = \| c(t) \|^2$ is constant.

$$\frac{d}{dt}\left(c(t) \cdot c(t)\right) = c'(t) \cdot c(t) + c(t) \cdot c'(t)$$
$$= 2c'(t) \cdot c(t) = 0$$

Thus $c'(t)$ is perpendicular to $c(t)$. $\square$

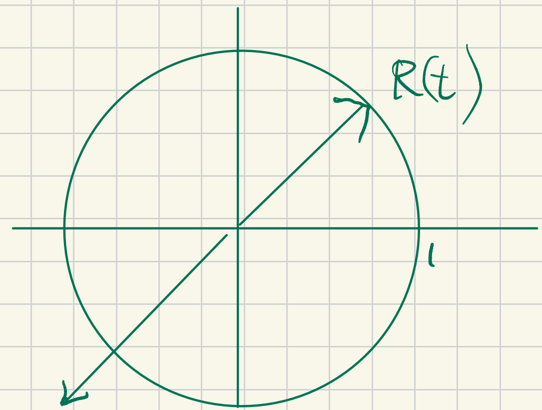
Example: find the force on a particle in circular motion, of mass 1, tracing a path $R(t) = (\cos t, \sin t)$

Or mass $= 3$, $R(t) = (\cos 2t, \sin 2t)$.

Solution. $r = R'(t) = (-2\sin 2t, 2\cos 2t)$

$a = R''(t) = (-4\cos 2t, -4\sin 2t)$.

Force $3(-4\cos 2t, -4\sin 2t$.



Force $=$ Centripetal force.

Like questions 13, 14, 23.

The acceleration, initial velocity and initial position of a particle are

$a(t) = (1,2,3)$, $v(0) = (2, -1, 1)$, $c(0) = (3,2,1)$.

Find $c(t)$.

Solution $a = v'$, so $v = \int a$

$= (t, 2t, 3t) + v(0) = (t, 2t, 3t) + (2, -1, 1)$

$c = \int v = \left(\frac{t^2}{2}, t^2, \frac{3t^2}{2}\right) + (2t, -t, t)$

$+ \text{constant} (= c(0))$

$= \left(3 + 2t + \frac{t^2}{2}, 2 - t + t^2, 1 + t + \frac{3t^2}{2}\right)$ $\square$